GUIDELINES FOR THE USE OF CONTROL CHARTS FOR CONSTRUCTION AGGREGATES

1. SCOPE

1.1 The purpose of this guideline is to outline the procedures and requirements for creating control charts based on construction aggregate Quality Control (QC) test results.

2. RELEVANT DOCUMENTS

2.1 ASTM Manual on Presentation of Data and Control Chart Analysis, MNL7, 6th Edition. April 1990.

2.2 International Standard ISO 7870 Control Charts - General Guide and Introduction

2.3 International Standard ISO 8258 Shewhart Control Charts

3. GENERAL

2.1 A control chart is a QC technique used to graphically illustrate the variation of a measured value of a product or process. This variation may be the result of chance causes or assignable causes.

2.2 Control charts are used to determine whether the product or process is *in control* through the application of statistical principles.

2.3 Control charts described by this guideline are limited to:

i) Control charts for individuals based on single values (X) and moving range (mR), designated as a **Type 1** chart; and

ii) Shewhart control charts for variables $\overline{X}$[1] (average, or mean) and R (range), designated as a **Type 2** chart.

Note 1: Acceptance of products on contracts is based exclusively on meeting the applicable Quality Assurance requirements. Control charts are used soley for the QC purpose of determining testing frequency as allowed by the appropriate specification and are independant of acceptance or rejection of the material.

4. DEFINITIONS

assignable cause — variation within a set of test data that is either random or systematic that occurs outside the stable pattern of chance causes

[1] The symbol $\overline{X}$ is read as **X bar** and the symbol $\overline{\overline{X}}$ is read as **X double bar**. The bar over any symbol always indicates an average - thus, $\overline{X}$, means an average of the X's.

central line	a value determined as the process average for the purposes constructing of a control chart.
chance cause	variation within a set of test data that is the result of attributes inherent in any stable system of production and inspection
control limit	a value based on 3-sigma (3σ) limits (root-mean-square deviation) of a normal distribution plotted on a Type 1 or Type 2 control chart. Control limits are determined by the subgroup size and average range ($\overline{R}$) of a set of data and may be either an Upper Control Limit (UCL) or a Lower Control Limit (LCL).
quality assurance	means a system or series of activities that are performed on manufactured or processed material in order to provide confidence that the product complies with applicable specification requirements.
quality control	means a system or series of activities that are carried out during the process or manufacturing of a product in order to ensure that it meets the intended specifications.
specification limit	a value for a given test method indicated by the appropriate specification or special provision that forms the basis of acceptance or rejection of the material. A specification limit may either be a maximum value (Upper Specification Limit USL) or a minimum value (Lower Specification Limit LSL)
subgroup	one of a series of groups of observations. Subgroups are selected so that each item inspected within the subgroup is as nearly representative of production as possible over a given period of time.
target value	an established process average determined from a minimum number of subgroups or test data

5. SYMBOLS

The following symbols are referred to by this guideline:

k	number of subgroups;
n	number of observed values within a data set or within a subgroup;
X	observed value of a measured characteristic. Specific values may be designated as $X_1, X_2, X_3, \ldots, X_i, \ldots, X_n$. ;
$\overline{X}$	average (arithmetic mean);
$\overline{\overline{X}}$	average of a set of values for $\overline{X}$. The value $\overline{\overline{X}}$ is also known as the grand average;
R	range of a set of observed values calculated as the difference between the largest and smallest value within a subgroup;
mR	moving range formed from successive pairs of observations. Adjacent moving ranges will have one value in common;

$\overline{R}$, $m\overline{R}$	average of a set of values for range R, and moving range mR;
A_2	a factor of $\overline{R}$ used to determine the distance from the central line to 3σ control limits on an $\overline{X}$ chart;
D_4	a factor of $\overline{R}$ and $m\overline{R}$ to determine the 3σ upper control limit on a chart for R, or mR respectively;
E_2	a factor of $\overline{R}$ to determine the 3σ control limits on an individual X chart.

6. OBJECTIVES OF A CONTROL CHART

5.1 Control charts for construction aggregates may be used for some or all of the following objectives:

a) to determine whether the aggregates can meet the given specifications;

b) to determine whether a change in aggregate production is required;

c) to determine whether a change in inspection procedures is validated;

d) to provide a basis for action whenever assignable causes of variation are indicated.

Note 2: When creating control chart, it is important that test data for a specific product and location within a source is collected separately. For instance, different benches within a quarry may yield aggregates with widely different physical properties. Also, different aggregate products, such as HL3, or HL8 stone may have widely different production properties. In order to produce a meaningful chart, test data for different aggregate products and/or from widely different locations within a source should not be included in the same data set.

7. TYPE I CONTROL CHART (X-mR)

7.1 See Example 1 for worked example of a Type 1 control chart.

7.2 A Type 1 control chart consists of plotting individual test results for X in order to identify the average value of the process, and trends in the individual test results. A moving range (mR) chart is used to display changes in the absolute difference between successive test results.

7.3 A plot of individual values for X is reduced to a simple run-chart with n being equal to the number of test results. Data for the mR chart are obtained from each successive pair of observations.

7.4 Individual test data from the appropriate test method are recorded in chronological order in an appropriate table.

7.5 Calculate $\overline{X}$ by summing the total of the each value in the data set and and dividing by the number of observations n:

$$\overline{X} = \frac{X_1 + X_2 + \ldots + X_i + \ldots + X_n}{n} = \frac{1}{n}\sum X$$

7.6 Calculate individual ranges R for successive pairs in the data set, starting with the first pair of data (X_1, X_2), by subtracting the lower value from the higher value:

$$R_1 = X_{max} - X_{min}$$

7.7 Similarly, calculate the range R between the next successive pair (X_2, X_3):

$$R_2 = X_{max} - X_{min}$$

7.8 Calculate the range R for each remaining pair in the data set.

7.9 Calculate the average moving range $m\overline{R}$ by summing the ranges of each successive pair and dividing by the number of observations minus one (n - 1):

$$m\overline{R} = \frac{R_1 + R_2 + \ldots R_i + \ldots + R_n - 1}{n-1} = \frac{1}{n-1}\sum R$$

7.10 Calculate the control limits for R using the factor D_4 from Table 1.

$$UCL_R = D_4\, m\overline{R}$$

$$LCL_R = 0$$

7.11 Calculate the control limits for the X chart by using the factor E_2 from Table 1.

$$UCL_X = \overline{X} + E_2\, m\overline{R}$$

$$LCL_X = \overline{X} - E_2\, m\overline{R}$$

7.12 Construct the X chart. Plot the central line as a solid horizontal line at $\overline{X}$. The control limits should be drawn as horizontal lines at the calculated values. The specification limit(s) given by the appropriate specification for the test method under consideration may be plotted on the same chart.

7.13 Plot individual values for X for each test result. Individual points are joined by a connecting line.

7.14 Construct the mR chart. Plot the central line as a solid horizontal line at $m\overline{R}$. The upper control limit should be drawn as a line at the computed value of UCL_R.

7.15 Individual values of R are then plotted for each subgroup in sequence. Individual points are joined by a connecting line.

8. TYPE 2 CONTROL CHART ($\overline{X}$-R)

8.1 See Example 2 for worked example of a Type 2 control chart.

8.2 A Type 2 control chart is based on the variation of mean and range within individual subgroups, or lots. For practical purposes, each subgroup k should contain 4 values such that n = 4.

8.3 Data from the appropriate test method, or in the case of gradation, LS-602, the appropriate sieve is recorded in chronological order.

8.4 Calculate $\overline{X}$ for each subgroup by adding each measurement of the subgroup and dividing by n:

$$\overline{X} = \frac{X_1 + X_2 + X_3 + X_4}{4} = \frac{1}{4}\sum X$$

8.5 Calculate the range for each subgroup by subtracting the lowest value from the highest:

$$R = X_{max} - X_{min}$$

8.6 Calculate the average range $\overline{R}$ by summing the ranges of each subgroup and dividing by the number of subgroups, k:

$$\overline{R} = \frac{R_1 + R_2 + \ldots R_i + \ldots + R_k}{k} = \frac{1}{k}\sum R$$

8.7 Calculate the control limits for R using the factor D_4 from Table 1:

$$UCL_R = D_4 \overline{R}$$

$$LCL_R = 0$$

8.8 Calculate $\overline{\overline{X}}$ by summing the $\overline{X}$ values and dividing by the number of subgroups, k:

$$\overline{\overline{X}} = \frac{\overline{X}_1 + \overline{X}_2 + \ldots + \overline{X}_i + \ldots + \overline{X}_k}{k} = \frac{1}{k}\sum \overline{X}$$

8.9 Calculate the control limits for $\overline{X}$ using the factor A_2 from Table 1.

$$UCL_X = \overline{\overline{X}} + A_2 \overline{R}$$

$$LCL_X = \overline{\overline{X}} - A_2 \overline{R}$$

8.10 Construct the $\overline{X}$ chart. Plot the central line as a solid horizontal line at $\overline{\overline{X}}$. The upper and lower control limits should be drawn as horizontal lines at the calculated values. Plot individual values for $\overline{X}$ for each successive subgroup. Individual points are joined by a connecting line.

8.11 Construct the R chart. Plot the central line as a solid horizontal line at $\overline{R}$. The upper control limit should be drawn as a line at the computed value of UCL_R. Individual values of R are then plotted for each subgroup number in proper sequence. Individual points are joined by a connecting line.

Table 1. Factors for Calculating Control Chart Limits[1]

Chart for Averages A_2	Chart for Ranges D_4	Chart for Individuals E_2
0.729	3.267 *(Type 1 chart)* 2.282 *(Type 2 chart)*	2.659

9. **INTERPRETING THE CONTROL CHART**

9.1 When a process is in statistical control, the individual points on a control chart (X, mR, $\overline{X}$, R) will fluctuate randomly within the control limits without any recognizable pattern. In other words, the

[1] From ASTM Manual on Presentation of Data and Control Chart Analysis, 6th Ed. *Table 6, Factors for computing control chart lines - no standard given*, pg. 57; and Table 50, *Factors for computing control chart lines - chart for individuals*, pg 92.

variation in the data is the result of chance causes that are inherent within the process. In general, a controlled process will meet the following criteria:

1) none of the points exceed the control limits;
2) all of the points fall randomly above and below the central line; and
3) most of the points are near the central line with only a few points spread out close to the control limits.

9.2 Whenever the variation indicated by the control chart is the result of causes that lie apart from those inherent in the process, several types of unnatural patterns may be observed. These patterns are indications that assignable causes are present. Assignable causes may be identified and corrected if they affect the cost of the process or the acceptability of the material.

9.3 Patterns indicating that assignable causes are present include the following:

a) Instability - points fall outside the control limits and/or the chart is characterized by erratic up and down swings between the control limits. Examples of assignable causes for this pattern include: continual adjustment; over adjustment; carelessness.
b) Trend - a series of consecutive points without a change in direction. Example of assignable cause: tool or equipment wear; small changes in source materials.
c) Cycles - short repeated patterns with alternate high peaks and low valleys. Example of assignable cause: seasonal effects; staff rotation.
d) Change in process average - a lack of control is typically indicated by runs of seven or more points in succession on the same side of the central line. Example of assignable cause: broken screen; large change in source materials; change in staff.
e) Freaks - this occurs only if sudden causes affect the process which results in a change in the sample. Example of assignable cause: incomplete operation; wrong setting or adjustment that is corrected immediately; non-representative sample; erroneous testing or calculation.
f) Hugging the central line - all points are near the central line.

Note 3: Any action taken as a result of the pattern of variation shown in the control charts is dependent on comparing the control chart with the applicable specification. It is beyond the scope of this guideline to identify all possible scenarios. The choice among the various actions is a matter of relative economy.

10. CONTINUED USE OF THE CONTROL CHART

10.1 Type 1 charts for physical or production characteristics of aggregates require 20 individual test results in order to establish future testing frequencies. When plotting test results for physical characteristics, it is sufficient to plot the most recent data only. Thus, when new test data is obtained, the oldest test data may be ignored and the control chart can be re-established based on the most recent set of 20 test results.

10.2 Type 1 charts for production characteristics, ie, gradation, may be dealt with differently because of the need to generate new test data for each separate contract and to avoid the tedious recalculation of the control chart after every set of test results. Initial calculation of control chart parameters based on a minimum of 20 values may be used to establish target values that can be used to monitor the variation of future production. Subsequent values may be plotted on the existing chart without recalculating a new average, $\overline{X}$. When using a Type 1 chart for gradation, the central lines and control limits should be revised periodically after every 10 or 15 samples. In this case, as for a Type 1 chart for physical characteristics, the control chart may be re-established based on the most recent set of 20 test results.

10.3 For Type 2 charts, a minimum of 20 subgroups should be examined before statistically valid target values can be determined. It should be noted that initial subgroups from the start of production may not be representative of continued production. As a result, control limits based on less than 20 subgroups are preliminary and should be regarded as trial control limits only. The analysis of trial control limits should be limited to determining whether past operations were in control i.e., control chart analysis should only be applied to the data used in their calculation. Provided that trial control limits show that the process in control, they should be modified as more subgroups are obtained in order to apply them to future production.

10.4 If the process has been in control according to averages and ranges, trial control limits may be extended to future production. Frequent monitoring of control charts at regular intervals so that meaningful and up-to-date control lines are maintained, e.g., weekly, monthly, every 20,000 tonnes of production etc.

10.5 In any case, after a process change control charts should be revised as soon as 20 samples under the new conditions have accumulated.

EXAMPLE 1. Worked Example for Type 1 Control Chart.

Aggregate type: Granular O

Test Method Name: Freeze-Thaw **LS-Number:** 614 **USL:** 15.0%

Sample	Test Result	Moving Range	Sample	Test Result	Moving Range
n	*X*	*mR*	*n*	*X*	*mR*
1	10.9	-	**11**	8.5	0.8
2	11.4	0.5	**12**	11.3	2.8
3	12.2	0.8	**13**	10.6	0.7
4	9.8	2.4	**14**	10.9	0.3
5	9.6	0.2	**15**	9.3	1.6
6	10.4	0.8	**16**	9.5	0.2
7	11.2	0.8	**17**	13.1	3.6
8	10.9	0.3	**18**	12.3	0.8
9	9.6	1.3	**19**	11.8	0.5
10	9.3	0.3	**20**	11.1	0.7
			Total (*n* = 20)	213.7	19.4

Calculations:

Calculate $\overline{X}$: $n = 20$

$$\overline{X} = \frac{213.7}{20} = 10.7$$

Calculate $m\overline{R}$:

$$\overline{mR} = \frac{19.4}{19} = 1.02$$

Calculate control limits for the mR chart:

$$UCL_R = D_4\ \overline{R} = 3.267(1.02) = 3.3$$

$$LCL_R = 0$$

Calculate the control limits for the X chart:

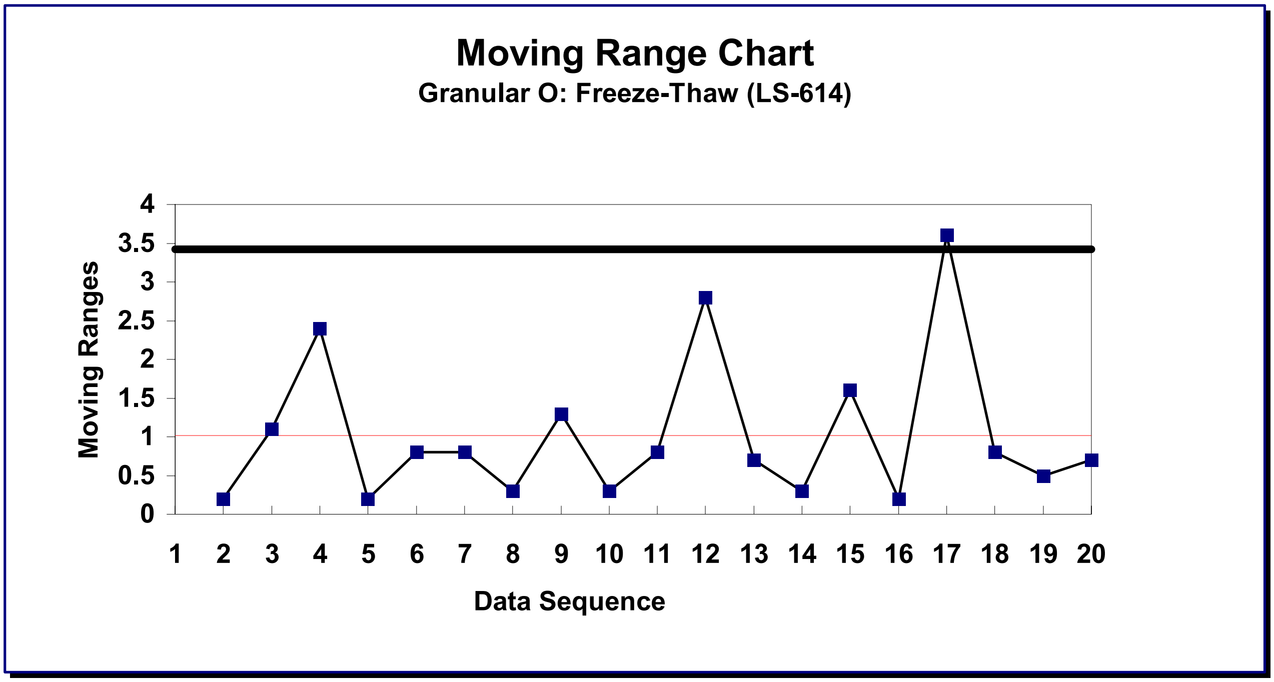
$$UCL_X = \overline{X} + E_2\ m\overline{R} = 10.7 + (2.659)\ (1.02) = 13.4$$

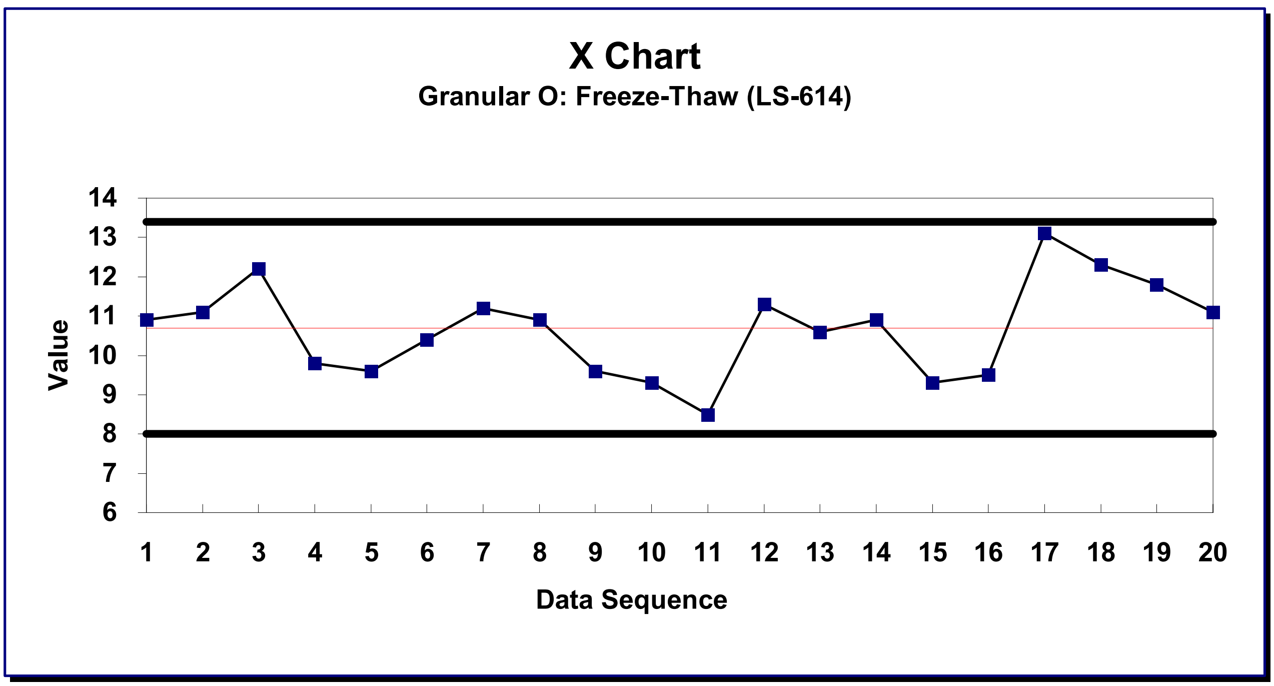
$$LCL_X = \overline{X} - E_2\ m\overline{R} = 10.7 - 2.7 = 8.0$$

Plot the control chart:

Each data point in the following charts is determined from individual test data.

Type 1 control chart:





The moving range chart is examined first. If this chart is not in control, then the values chart cannot be in control. In the above example, the range at data point *17* lies outside the 3σ limits for statistical control. This indicates that an assignable cause for the variability is present. For conditions such as these, the cause may be investigated and, if found, corrected. Identifiable causes that have been properly documented and eliminated, may justify the removal of the data and recalculation of the control chart. If the cause cannot be found, the data must be included in all subsequent recalculations of the control chart.

Note 4: In these examples, all products meet the appropriate material specifications for acceptance.

EXAMPLE 2. Worked Example for Type 2 Control Chart.

Record Sheet for $\overline{X}$ and R Control Chart

Aggregate type: Granular A

Test Method Name: Gradation **Number:** LS-602

Sieve: 75 µm **USL:** 8.0% **LSL** 2.0% **Maximum Range:** 5%

Subgroup (Lot)	Individual values (n = 4)				Average	Range
K	**1**	**2**	**3**	**4**	$\overline{X}$	***R***
1	7.3	7.8	7.0	7.9	7.5	0.9
2	7.8	7.7	7.2	6.4	7.3	1.4
3	6.7	6.6	7.2	6.4	6.7	0.8
4	5.5	7.2	7.1	6.5	6.6	1.7
5	7.1	6.2	5.4	6.6	6.3	1.7
6	6.9	7.1	7.3	7.1	7.1	0.4
7	7.0	7.0	6.9	6.9	7.0	0.1
8	6.4	5.2	4.8	4.7	5.3	1.7
9	6.2	6.8	6.8	6.1	6.5	0.7
10	7.3	4.3	7.5	4.0	5.8	3.5
11	6.0	6.1	6.4	3.7	5.6	2.7
12	4.9	3.2	4.5	4.7	4.3	1.7
13	4.4	6.7	4.1	4.6	5.0	2.6
14	4.0	4.4	6.0	4.6	4.8	2.0
15	4.1	4.1	3.4	3.3	3.7	0.8
16	3.6	5.1	3.9	6.6	4.8	3.0
17	3.2	4.3	4.4	3.3	3.8	1.2
18	4.9	4.1	4.5	4.9	4.6	0.8
19	6.0	4.6	4.4	5.1	5.0	1.6
20	3.8	4.6	4.2	6.7	4.8	2.9
TOTALS *k = 20*	-	-	-	-	**112.3**	**32.2**

Calculations:

Calculate $\overline{\overline{X}}$*:*

$$\overline{\overline{X}} = \frac{112.3}{20} = 5.62$$

Calculate R:

$$\overline{R} = \frac{32.2}{20} = 1.61$$

Calculate control limits for the R chart: $UCL_R = D_4\ \overline{R} = 2.282(1.61) = 3.67$ $LCL_R = 0$

Calculate the control limits for the $\overline{X}$ *chart:*

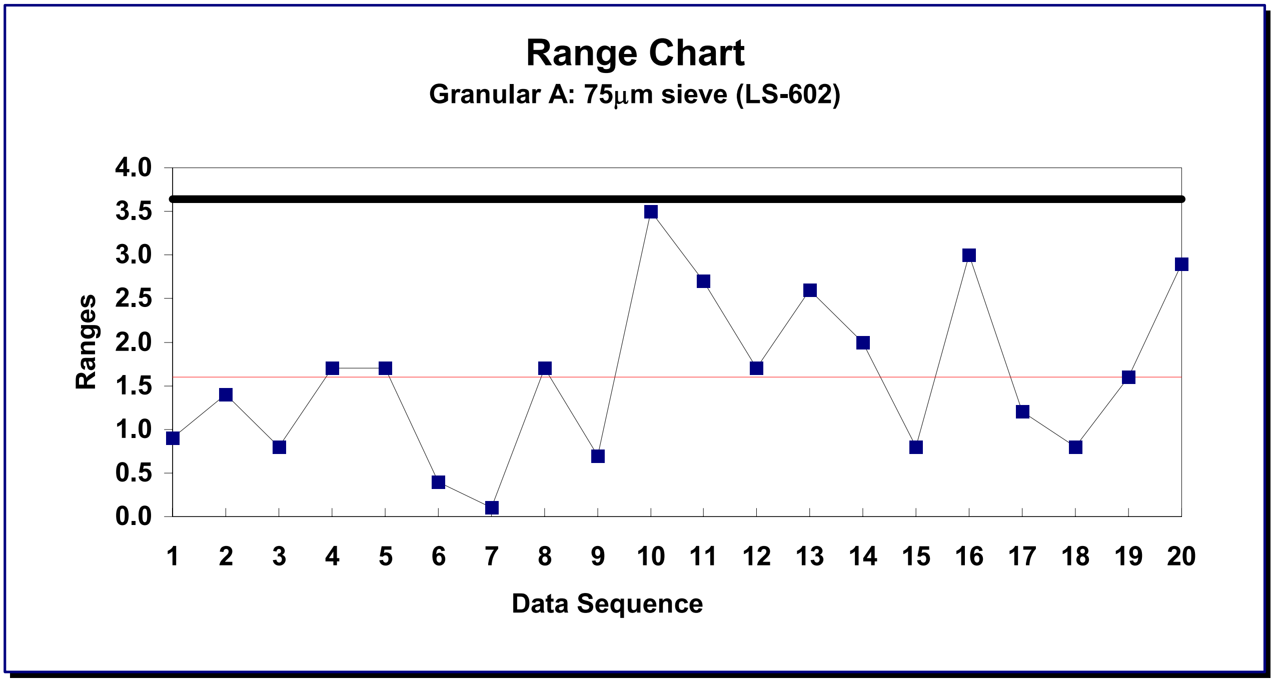
$$UCL_X = \overline{\overline{X}} + A_2\overline{R} = 5.62 + (0.729)(1.61) = 6.79$$

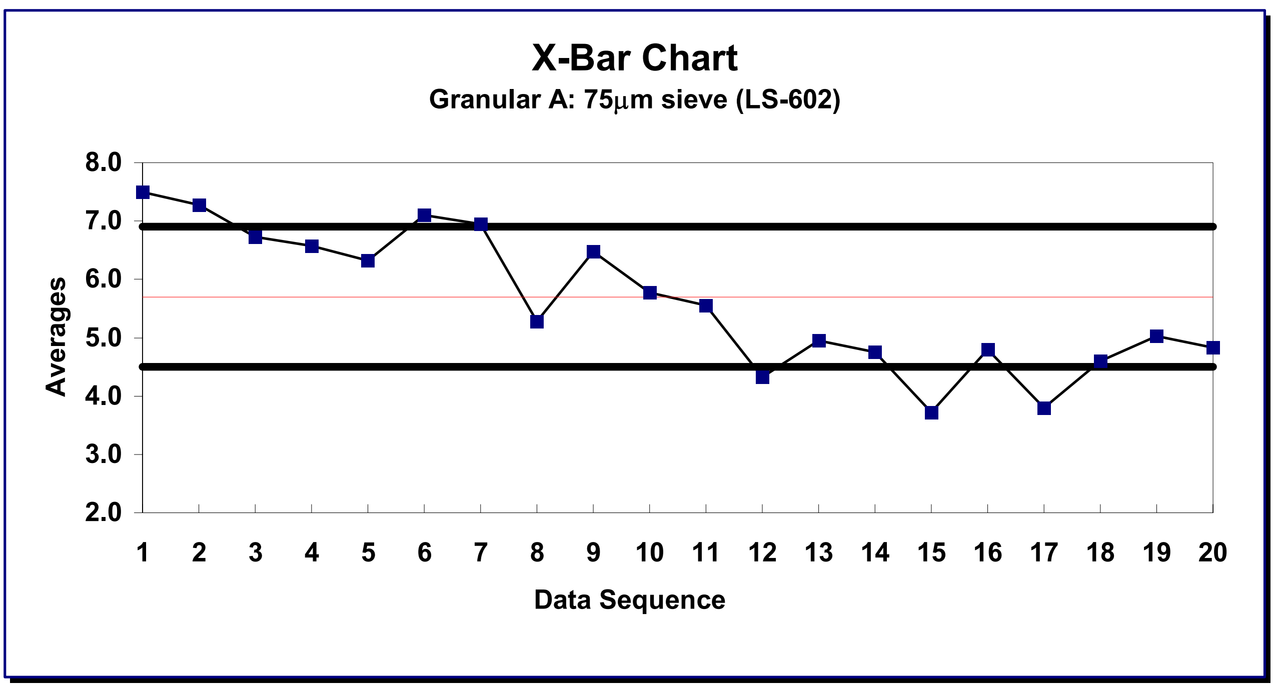
$$LCL_X = \overline{\overline{X}} - A_2\overline{R} = 5.62 - (1.17) = 4.45$$

Plot the control chart:

Each data point shown on the following graphs represents the individual statistics (average, range) from a subgroup of 4 test results.

Type 2 Control Chart





In this example, the range chart shows that the range variation between each subgroup is in *statistical control*. However, the X-bar chart shows that the process is significantly *out of control*. The data shown here is indicative of a process that is undergoing progressive changes and has not yet achieved any consistency in either the material, the production, or the testing of the product. Assignable causes for this case are most likely simple to identify. If they can be isolated, documented, and eliminated, the data which represents the remaining stable process may be recalculated in a new control chart.